**Pen and Paper Analysis of Probability Problems**

If you have a fair five-sided die with the numbers 1,2,3,4, and 5 on its faces each number will have a 20% chance of being rolled. This is because there are 5 total outcomes, so any number only has a 1/5 chance of being the outcome. If you roll two of these dice, there are 25 total outcomes with only 8 unique outcomes pictured below.Graphical user interface, application, table, Excel

Description automatically generated

The table on the right indicates the likelihood of each sum appearing. The chances start at 4% for an outcome of two (which is only possible if you roll snake eyes) and increase by 4% for each successive outcome. It peaks at 20% for a sum of 6 which is always possible no matter what your first roll is. Then it decreases by 4% until the last sum, 10, which is only possible with two fives. Console screenshot for verification:

Graphical user interface, text

Description automatically generated

A fair eight-sided die with the faces 1,2,3,3,4,5,5, and 5 displays different probabilities than the five-sided die. Any face on a fair eight-sided die has a 1/8 chance of appearing. To calculate the odds with our custom faces, we simply multiply the number of times the face appears on the die by 1/8. There is only one 1, so there is only a 1/8 (12.5% chance of a 1 being rolled). The same goes for the 2 faces. There are two 3 faces so a three should appear twice every eight roles (2/8 or 25%). There is only a singular 4 which is 1/8 or 12.5% again. Finally, there are three 5 faces, meaning a 5 has a 3/8 chance of appearing (37.5%). When rolling two of these dice and summing the outcomes, there are 8 unique outcomes (sequence from 2-10) present in the 64 possible outcomes. (Percentages shown were rounded at the 3rd digit).

Table, Excel

Description automatically generated

The explanation of the calculation for the first time still applies. You count the number of times an outcome appears and divide it by the total number of outcomes to get a percentage chance. Interestingly, the most likely outcome for these dice is 8 and not 10 like I suspected it would be. That’s because 4 of the 8 faces in your first role still allow an 8, whereas only 3 faces allow for a 10. Console verification:

Graphical user interface, text

Description automatically generated

The four dice problem can be represented with six tables.

* D1: 1,2,3,9,10,11
* D2: 0,1,7,8,8,9
* D3: 5,5,6,6,7,7
* D4:3,4,4,5,11,12

Two determine the probability of which dice will roll higher for any pair, you simply count up all of the outcomes either dice will win out of the 36 possible outcomes. Notably, if you want the probability when the order of the rolls is switched, you simply swap the percentages. For the below table, if the first roll is the first dice and the second roll is the second dice, the first dice will be higher 61.11% of the time. If you invert the order, the first dice will be higher 33.33% of the time.

Graphical user interface, application, table, Excel

Description automatically generated

Tables for the other comparisons follow:

Table

Description automatically generatedApplication, table, Excel

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Table, Excel

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Table

Description automatically generated

Application, table

Description automatically generated

Console verification (assume the left column is the first roll, the percent is the chance it’s higher):

A screenshot of a computer

Description automatically generated with medium confidence